# Probability

# **Deduction vs Induction**

- Deduction is concerned with *necessary* reasoning. A deductive argument is "good," i.e. sound, because the truth of the premises guarantee the truth of the conclusion.
- Induction is concerned with *probabilistic* reasoning. An inductive argument is "good," i.e. cogent, because the truth of the premises makes the truth of the conclusion very likely.

# Probability – Three Theories

- Classical or A Priori
- Relative Frequency
- Subjectivist

A Priori Theory

Ascribes to a simple event a fraction between 0 and 1.

- Relies on two assumptions:
  - 1. All possible outcomes of a given situation can be determined. 2. Each possible outcome has an equal probability of occurring.

#### Pr(A) = f/n

The probability of an event A occuring is equivalent to the number of favorable outcomes f divided by the number n of possible outcomes

"What is the probability that an even number will come up on any toss of the die?" Three positive outcomes (2, 4, and 6) The probability is 3/6, or 1/2.

## **Relative Frequency**

The theory that some probabilities can be computed by dividing the number of favorable cases by the total number of observed cases.

#### Pr(A) = p/tn

The probability of an event, Pr (A), is equal to the number of positive outcomes, p, divided by the total number of observed cases, tn.

# Subjectivist

- The theory that some probability determinations are based on the lack of total knowledge regarding an event.
  - In every subjectivist calculation the following are important:
  - Reliance on experience
  - Availability of relevant information





# **Probability Calculus**

- The branch of mathematics that can be used to compute the probabilities of complex events from the probabilities of their component events.
- Our time is limited, so we will only be looking at some basic rules that emerge from the probability calculus

- **Independent** Events are said to be independent if the occurrence of one event has no effect on the probability that the other event will occur.
- **Mutually Exclusive** If one event occurs, the other event cannot occur.

## **Conjunction Methods**

• **Restricted conjunction method:** The method that is used in situations dealing with two or more independent events, where the occurrence of one event has no bearing whatsoever on the occurrence or nonoccurrence of the other event.

#### $Pr(A and B) = Pr(A) \times Pr(B)$

• General conjunction method: The method that is used for calculating the probability of two or more events occurring together, regardless of whether the events are independent or not independent.

 $Pr(A and B) = Pr(A) \times Pr(B, if A)$ 

## **Disjunction Methods**

• Restricted disjunction method: The method that is used when two (or more) events are independent of each other, and the events are mutually exclusive.

### Pr(A or B) = Pr(A) + Pr(B)

• General disjunction method: The method that is used for calculating the probability when two or more events are not mutually exclusive.

 $Pr(A \text{ or } B) = [Pr(A) + Pr(B)] - [Pr(A) \times Pr(B)]$ 

## Negation Method

 Negation method: The method that is used once the probability of an event occurring is known; it is then easy to calculate the probability of the event not occurring.

#### $Pr(\sim A) = 1 - Pr(A)$

Example: The restricted conjunction method gives the probability of getting two tails in successive tosses of a coin as 1/4. Negation formula gives the probability that two tails in succession will *not* occur:

#### Pr (~A) = 1 - 1/4

The probability that two tails will not occur in succession is:  $1 - \frac{1}{4} = \frac{3}{4}$ 

## True Odds in Games of Chance

- In a "fair" game of chance, the payoff odds of winning guarantee that, in the long run, the gambler will break even.
- The ability to calculate true odds allows you to determine whether you are playing a fair game or whether the odds are stacked against you.
  - Casino example: If you decide to place your bet on your lucky number, what will happen? The casino is willing to give you 35:1 odds for this bet. But since there are 38 numbers, the odds are once again not true. Odds have to deviate only slightly away from true odds to provide the casino with a winning margin, especially with millions of bets taken annually.

## **Bayesian Theory**

- **Thomas Bayes** advanced probability theory by uniting the *probability calculus* and *relative frequency theory* into a method for calculating conditional probability.
- **Conditional probability:** The calculation of the probability of an event if another event has already happened.

 $Pr (A, if B) = \frac{Pr (B, if A) \times Pr (A)}{Pr (B)}$