Section 1.1: Basic Logical Notions

**Definition:** An *argument* consists of
- A set of sentences known as the *premises*, and
- A single sentence, known as *conclusion*.

**Comment:** The sentences in an argument must all be *declarative* sentences, that is, sentences capable of being true or false, as opposed to imperatives (commands) like “Do your homework” and interrogatives (questions) like “Is it raining?”

**Definition:** An argument is *valid* if and only if it is necessary that, if all of its premises are true, then the conclusion is true as well.

Alternatively (and equivalently):

**Definition:** An argument is *valid* if and only if the following situation not possible: that the premises of the argument are true while the conclusion of the argument is false.

**Comment:** The idea behind these (equivalent) definitions is that, in a valid argument, the truth of the premises guarantees the truth of the conclusion. That is, it is not enough (for validity) for the premises, say, to make the conclusion highly probable, or beyond reasonable doubt. Rather, validity in the above sense (or *deductive* validity, as it is often called) is a purely logical notion. For an argument to be valid in the above sense, the connection between premises and conclusion must be so strong that it is not even possible, in the strongest sense, that the premises be true and the conclusion false -- you must not even be able to conceive of circumstances in which that happens.

**Examples of Valid Arguments**

Every democrat is a liberal.
Bill is a democrat.
Therefore, Bill Clinton is a liberal

No true cyclist has hairy legs.
Everyone with hairy legs listens to country music.
Therefore, no true cyclist listens to country music.

**Example of an Invalid Argument**

Every democrat is a liberal.
Bill is a liberal.
Therefore, Bill Clinton is democrat.

**Comment:** It is very important to realize that validity *does not require true premises*. The definition of validity only requires that if the premises are true, then the conclusion must also be true. Thus, to
evaluate an argument for validity, try to imagine a situation -- no matter how outlandish, as long as it is logically possible -- in which the premises are true but the conclusion is false. If you cannot do it, then the argument is valid. (This test is still not really precise, since what one can or cannot imagine is a bit wooly. To get really precise, we'd have to start doing what logicians call model theory, which we begin to introduce in Chapter 4 (and which we may or may not get to in this class). But these informal notions will do well enough for our purposes here.)

Comment: Notice that, as a consequence of the definition of validity, if it is not possible for the premises of an argument all to be true (that is, if they are contradictory) then the argument is by definition valid, since it is not possible for the premises all to be true and the conclusion false.

No US president is libidinous.
Bill Clinton is a US president.
Bill Clinton is libidinous.
Therefore, Bill Clinton is a republican.

Comment: Notice also that if an argument has contradictory premises, it does not matter what the conclusion is. This is a feature of classical logic: anything follows from a contradiction. This may seem odd at first, but in fact you already have a sense that it's true. Suppose someone gives you an argument for a conclusion you find absurd. Then you might respond ``If that's true, then I'm a monkey's uncle.'' (Well, your grandfather might have responded that way.) The intuition that is reflected in this response is that, if something absurd were true, then anything would be true (and hence that the absurdity in question must be false). That is exactly what is captured in the logical principle that anything follows from a contradiction.

Definition: The premises of an argument are said to entail the conclusion of the argument if and only if the argument is valid.

Comment: This definition does not mean that validity and entailment are the same thing. Validity is a property of arguments as a whole, whereas entailment is a relation that holds between the premises and the conclusion of a valid argument.

Definition: An argument is sound if and only if it is both valid and all its premises are true.

Comment: As should be clear from the example above, it is soundness, not validity alone, that makes for a good deductive argument. If an argument is valid, you only know that if its premises are true, it's conclusion must be true also. So if you believe the premises of a valid argument are all true, i.e., if you believe it is sound, then you are rationally bound to believe the conclusion as well.